scottish music review

On the Ontology and Epistemology of Musical Intervals … or Bye Bye Pythagoras.

Graham Hair and Richard Parncutt.

1: A practitioner's story.

We begin this reflection with accounts of two specific, practical, personal experiences. Despite their anecdotal nature, they sum up situations which have led us to reflect on the general topic of musical intervals: in particular about the ontology of musical intervals (what exactly it is that we consider we know them to be) and about the epistemology of musical intervals (how we know what we know). We shall then attempt to relate such specific, practical and personal experiences to research by practising musicians and by scholars in various branches of musicology, and indeed research from the perspective of other disciplines which have something illuminating to say about the nature of musical intervals. Naturally, our attempt to illuminate such a fundamental topic must needs refer to historical attempts by many generations of scholars, scientists and practitioners, extending back — if "extending back" means as far back as Pythagoras (circa 569 – circa 475 BC) — over two and a half millennia.

Our first account comes from the perspective of composition and performance. It summarises our interpretation of the music-theoretical fallout from an overview of a couple of excerpts from a recent series of compositions and performances by our first author (GH). The compositions in question are a series of six duets for two singers and digital harmonium. The choice of the digital harmonium as the accompanying instrument was made partly just because, like all keyboard instruments, the tuning of every pitch is fixed precisely in advance — computer-controlled in fact in this case — but especially because the scale used in three of the duets was the 19-EDO scale (the scale with nineteen equal divisions of the octave), which is not generally available on more conventional instruments, while, in the other three songs, the conventional, ubiquitous 12-EDO scale (ie the scale with twelve equal divisions of the octave) was used, and the digital harmonium makes switching between the two scales trivially easy (which is not true of conventional keyboards). On the other hand, when we talk about tuning with respect to the two vocal melodies, we would probably be more accurate to speak of intonation, because every note singers sing is determined by moment-to-moment choices and decisions of the performers, and probably involuntary physical reactions, including such phenomena as "muscle-memory", and indeed their intonation varies not only by the note, but over the course of the duration of the note as well. Moreover, as every singer knows, the pitch of every note — onset, middle portion and termination — is influenced by the pitch, rhythm and other characteristics of the notes of her melodic line which precede and follow, and by the notes which are being sounded simultaneously or in proximity by other instruments and voices, and perhaps by other considerations

as well. Such moment-to-moment control of intonation is characteristic of stringed woodwind and brass instruments too, of course, but singers provide the locus classicus.

To return to the digital harmonium for a moment, there were also cultural reasons for the choice, as well as the harmonium's precise 19-EDO tuning, namely an element of "world music" sensibility, and particularly of Qawwali music. Qawwali is the sufi genre which features the harmonium — an instrument of European origin which musicians of the South Asian sub-continent have made completely their own in recent generations, albeit in a portable form uniquely of the sub-continent. But as far as this particular project goes, in the pieces from which discussion proceeds here, the juxtaposition of voices and digital harmonium also serves the particular purpose of exploring empirically aspects of the ontology and epistemology of melodic and harmonic intervals from a twenty-first century practical perspective.

The six duets are from a cycle of Sufi Fragments, to short texts (couplets extracted from longer, multi-couplet ghazals or ghazal-like poems) by the medieval Turkish Sufi folk poet Yunus Emre (1238-1320) in English translations by Suha Faiz, a Turkish Cypriot, born in Cyprus in the days of the British administration there, but who was sent to England for his education as a youth, and subsequently spent most of his life as a civil servant in the (then) British Foreign and Commonwealth Office (Faiz 2007). The reasons for choosing to start with a consideration of excerpts from this song-cycle was, as mentioned above, the use of both 12-EDO and 19-EDO scales, but also that each song is very short: in fact, just one minute long (restricting the material to be considered to manageable proportions). Nevertheless, despite the different scales, the two sets of duets share some chord-structures which any musician can easily recognise as perceptually similar, and the characteristic idiom of the two sets of duets is not recognisably different. For example: the harmony consisting of the pitchclasses GCDF (sometimes known in popular music circles as the "suspended dominant" chord), is used in both sets of songs. Likewise several other harmonies — also chords often employed in contemporary folk-derived, jazz ,"world music" and other styles of contemporary popular music — including ACE (the minor triad) and GBD (its inversion), CEGA (the "added sixth"), CDEG (the "added second") and AGFD (its inversion), GBDF (the "dominant seventh") and AFDB (its inversion), GBDFA (the "dominant major ninth"), and GBFAC♯, (the "dominant seventh/sharp eleventh": sometimes called the "dominant thirteenth"), as well as GCDF (the aforementioned "suspended dominant" chord). There are of course alternative nomenclatures for these chords, including (in respect of popular music, folk-derived, jazz, "world music" and related genres): a, C, Gsus7, G7, C6, G9, G7♯11, Bdim7 etc, and (in respect of contemporary musics of more "classical" orientation) the so-called "prime forms" of set-theoretical notation: 037, 0358, 0247, 0258, 0257, 02469 and 02468. The present author tends to utilise the last of these nomenclatures, because of his preference for their greater degree of neutrality or ambiguity in respect of such features of historical and popular musics as roots of chords and tone-centres.

The harmonic vocabulary of the Sufi Fragments is based on extensive deployment of these seven chords and various transformations (transpositions, inversions) of them. These transformations include transpositions to every degree of the chromatic scale, as well as inversions of the chord and their transpositions to every available scale-degree, in cases where the process of inversion generates different sets of pitchclasses from any of the transpositions (as with the "added second" chord, and the triad). Figure 1 lists this vocabulary of 120 chords for the 12-EDO scale.

In the 19-EDO scale, the same chords and intervals can be replicated (to within scalar variation), except that the interval we identify as a major second is 189 cents rather than 200, the minor third

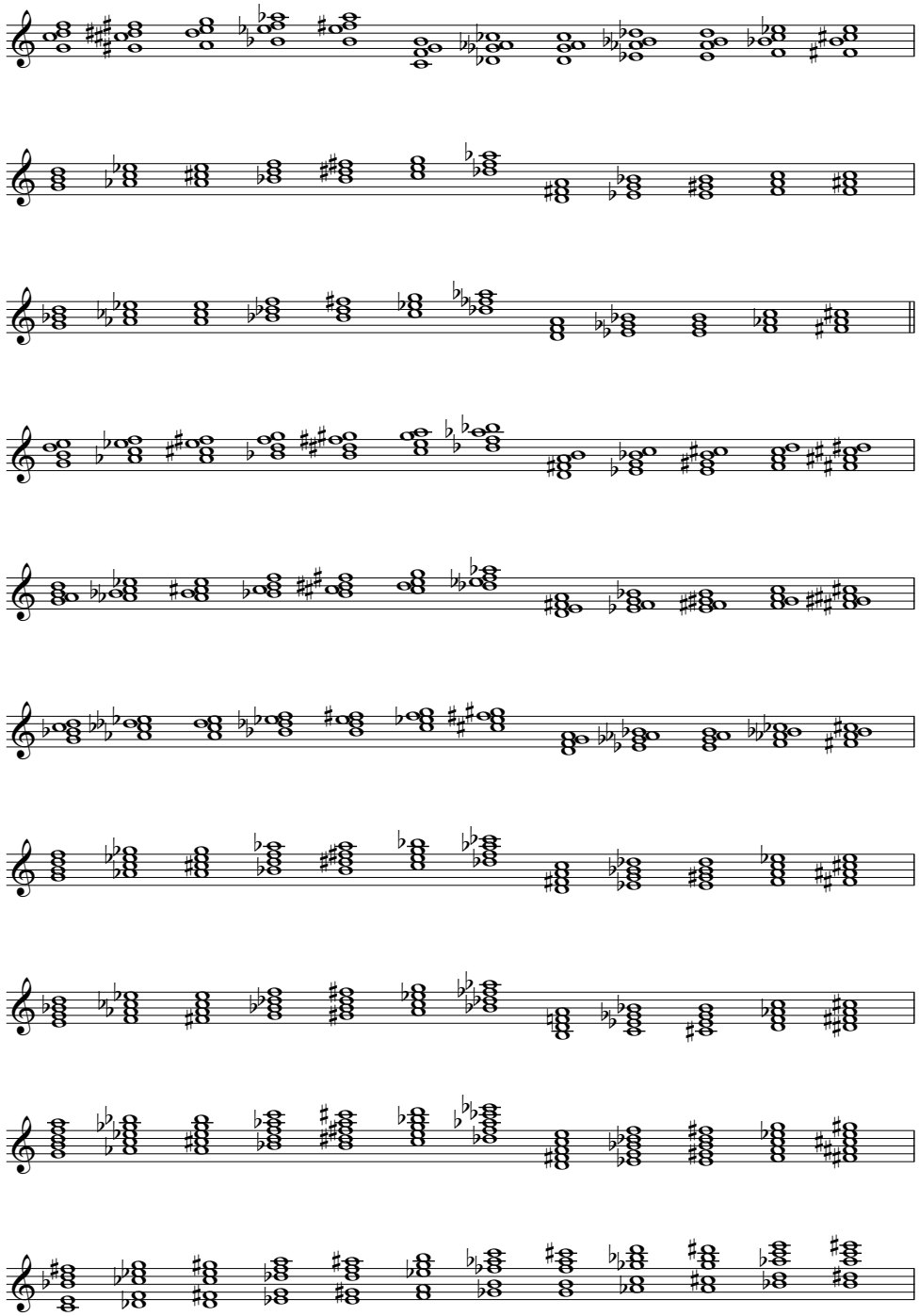

Figure 1: Chords for Sufi Fragments by Graham Hair

is 316 rather than 300, the major third is 379 rather than 400, the perfect fourth 505 rather than 500, and the diminished fifth 632 rather than 600, and except that there are 19 chords in each of the 10 rows of the chord-vocabulary chart listed in figure 1, rather than 12, ie 190 chords altogether. Sufi Fragments are miniatures, so only a selection of these 310 (120+190) chords can appear, but in compositions by the same composer of greater extent and scope, which share more or less the same compositional idiom, all or virtually all of these 120 or 190 chords often appear somewhere in the course of the piece.

This observation points to the fact that the underlying scale structure of Sufi Fragments is definitively the chromatic scale — whether 12-EDO or 19-EDO — rather than an underlying scale of 6 or 7 notes modified by "modal inflection" (ubiquitous flattening or sharpening of scale-degrees in a context governed essentially by Common Practice harmony, with its scale-degree-related functionality): modal inflections which, in toto, might well result in the deployment of all 12 or all 19 pitchclasses over the course of a piece. Nevertheless, a lesser number of pitches are usually in play in any 3- or 4-measure chunk of Sufi Fragments: what Dmitri Tymoczko Tymoczko (2011) calls "limited macro-harmony", the pitchclass collections in play at an intermediate level (ie above that of the individual chord and its associated scale: the level of about 3 or 4 measures), but below that of the pitchclass collections deployed overall in the piece (the level of 20-35 measures, in the case of each of the Sufi Fragments). Such limited macro-harmony is partly due to the relatively slow-moving harmonic rhythm of Sufi Fragments: contrasting with those idioms (eg twelve-tone) where the macro-harmony (pitchclass-turnover at the 3- or 4-measure scale) is essentially similar to the harmonic vocabulary of the piece as a whole (ie pitchclass turnover at the 20-35 measure scale).

To consider one interval, the major second between C and D: an interval which, when deployed harmonically (or indeed melodically) within the context of the 12-EDO scale, is 200 cents in size. When the major second is deployed within the context of 19-EDO scale, the interval is 189 cents. We can report, from personal experience, that the singers have no trouble at all in acclimatising themselves to the interval of the major second whichever of the two scalar contexts is the relevant context at the time. Indeed they barely notice the difference, because in each case, although the accompanying music in the digital harmonium part is either precisely 200 cents or precisely 189 cents, the singers' rendition is adaptable: approximately 200 or approximately 189 in each case. We would be better to say that, as far as the singers are concerned, the interval of a major second is approximately 194.5 ± 5.5 cents. Indeed, it's quite possible that, in another performance, the rendition of the major second would be a little less than 189 or a little more than 200 cents: perhaps 194.5 ± 10 cents would not be unreasonable. We reason, from such experiences, but also from other experiences which we shall describe in due course, that — essentially — intervals are approximate and that singers can easily adapt to a variety of sizes in the appropriate context (albeit not to absolutely any size whatever!), and still recognise that interval as a major second. That context might be cultural (the interval-sizes to which they are accustomed) as well as perceptual (the ability of the human perceptual apparatus to employ categorical means of sensing the size of any interval, and to disregard discrepancies between different representations of the major second — or any given interval — when it seems to the singer to make musical sense to do so).

It is also true, of course, that the category "major second" is often (usually?) considered to include the interval between the 8th and the 9th partials of the harmonic series. In order to accommodate this interval (204 cents) in the same category ("major second") which singers will render as "more or less 204 cents" we will need to expand our definition to something like "$194.5 \pm$ about12 cents":

a possible range from a few cents less than 189 to a few cents more than 204. Indeed, the category "major second" is also usually considered to include the interval between the 9th and 10th partials of the harmonic series, which, although smaller than the interval between the 8th and 9th partials, is still an interval which singers will normally identify as a major second: its size is approximately 182 cents, and including the distance of a few cents less than 182 to a few more than 204 in our definition means extending it to encompass a range of about ±15 cents: a range of about 30 cents altogether.

It is additionally true that the interval of a major second, when deployed within the context of contemporary folk, jazz and "world music" idioms (and within the Sufi Fragments, where a form of what Tymoczko Tymoczko (2011) has called the "Extended Common Practice" is intended, ie encompassing the Western Classical music tradition from about 1016 to 2016) is regarded as, and treated as, a consonant interval. This probably has something to do with the fact that these styles are based on concepts of consonance in which the consonant class of chords include a few of those with four pitchclasses, whereas most of the music of the tradition we call the Common Practice of the Western Classical Tradition — ie during the 300 or so years from about the mid-17th century to about the mid-20th — consonances are considered to be of three pitchclasses, and the four-pitchclass chords which are deployed (dominant 7ths, minor 7ths, half-diminished 7ths etc) are considered dissonances. The point here, however, is not that the concept of consonance has shifted somewhat (but not totally!) over time, but that — perhaps — the range over which the interval of a major second is considered to vary (about 30 cents) may have something to do with the fact the the major second in the aforementioned contemporary styles is considered to be a consonance. In the case of those chords of four or more pitchclasses deployed in contemporary folk, jazz and "world music" idioms and which are considered dissonant (broadly speaking: chords beyond triads, added seconds and sixths, suspended dominants and perhaps a few others), it is our opinion that the range of variation may therefore be even greater than 30 cents.

In a well-known treatise Parncutt (1989) — which draws on modern theories of perception as well as on historical music-theoretical precedent — our second author (RP) has noted — following Helmholtz (1863, 1998) — that the extent of sense of consonance of any given interval is due to the extent of the coincidence of the partials of its component notes. Thus, if one considers the first 10 items in the harmonic series formed on each of a pair of pitchclasses — for example the major second C and D — 3 of the 10 items (C D and E) are shared to within a close degree of approximation. C provides partials 1, 2, 4. and 8 of the series on C and the 7th partial of the series on D. D provides partials 1, 2, 4 and 8 of the series on D and the 9th partial of the series on C. E provides partials 5 and 10 of the series on C and the 9th partial of the series on D. Because the items in an harmonic series are related by ratios between the frequencies, this sense of coinciding partials may give rise to the idea that intervals may be defined in terms of ratios, in other words that — for example — major seconds defined in terms of the interval between the 8th and 9th partials of an harmonic series or between the 9th and the 10th partials, viz intervals of 204 or 182 cents, are somehow better or more precise major seconds than major seconds consisting of 189 or 200 cents. But this is not the usual situation with regard to the actual practice of musical performance, where, as our experience shows, the intervals of a major second (and indeed other categories of interval) can vary considerably, according to a much greater range of performance considerations.

The importance we sometimes attach to defining the interval between C and D as a ratio is, we suggest, a by-product of the alignment of partials, or, in other words, a function of the major second

within the context of a certain degree of consonance. If one considers ratios in which numerator and denominator are large enough, there are obviously several or many ratios which could be taken as definitions of the distance of a major second, not just 9:8 or 10:9, and given any distance in cents, one can always find a ratio which approximates to it, and the definition therefore needs to be modified to identify the smallest-integer ratio (9:8), despite the wide variety of the embodiments of the major second in non-keyboard performance situations, and that we have no difficulty at all in recognising these as "major seconds" as commonly as with any keyboard embodiment, 9:8 or modified.

There is, however, other reason for starting this article with a focus on the interval of a major second, in addition to the fact that the major second is a component interval in all of the four-note consonances deployed in contemporary folk, jazz and "world music" idioms (and indeed within the harmonies of the Sufi Fragments: namely, that the interval of a major second is the most ubiquitous melodic interval in musics worldwide (Vos and Trost 1989:383-396), so it makes sense to begin by asking questions about the extent and limits of its intonation. The reason for the term "major second" implies indeed that, although in Sufi Fragments, occurences of the intervals of a major second happen within a context in which the 12-EDO and 19-EDO chromatic scales are fundamental, many musics worldwide deploy the major second as statistically the most common interval between the notes of their melodies within a context in which scales of about 6 or 7 notes are fundamental. Indeed, the major second is called a second because a second is the most common interval between the adjacent notes in such melodies.

We might now ask whether there are circumstances under which singers — and listeners more generally — are especially attentive to the coincidence of component partials. One that comes to mind is the music of David Hykes's Harmonic Choir, which is characterised by extremely slow harmonic rhythm, in which the frisson produced in listeners by the coincidence of partials is a special feature (Hykes 2010). One might compare this situation to that of a violinist tuning her instrument, where, characteristically, she plays her open strings A4 and E5 over and over for maybe 10-12 seconds, listening carefully until she senses the coinciding partials to the strongest possible extent. If one again takes the first 10 partials of each of the notes A4 and E5, this scenario means that 8 of those 20 partials consist of the pitchclasses E and B. In such a scenario, the interval between A and E — whose designation as a "perfect fifth" reflects the influence of the 6- or 7-note scale — is likely to approach the ratio 3:2 (702 cents, again plus or minus a few cents, albeit in this case, "few" probably means less than 30). Another comparison, having in mind the history of music theory, might be with the historical device known as the monochord (Wikipedia: "an ancient musical and scientific laboratory instrument") designed so that a comparison could be made between vibrating strings of different lengths and the pitches of the resulting sounds, and the relationships between length and pitch remarked.

Easley Blackwood uses the term "recognizable" to describe the intervals and scales he investigates in his book about the tuning of intervals, scales and harmonies (Blackwood 2014), where he concludes that even the category "perfect fifth" (a fundamental interval of most tonal systems, and one usually considered, under conventional wisdom, to be the interval to whose precise tuning human beings are most sensitive) is indeed "recognizable" over the range 685 to 720 cents, and he composed a whole series of works for electronic keyboards in which the keyboards were tuned to all the available equal divisions of the octave from 13-EDO to 24-EDO, in which a huge variety of different representations of many different intervals familiar from other repertoires is embodied.

But the circumstances of the Harmonic Choir's performances, of tuning a violin, and of ex-

perimenting with a monochord are all special — extreme — performance situations. They have limited application to the situation in "normal" musical performances, especially those where the moment-to-moment intonation is under the control of the performer, ie in the playing of string, woodwind and brass instruments, and — above all — in singing, rather than in performances by keyboard and keyboard percussion instruments. This is the motivation for our distinction here between tuning (applying particularly to keyboards, where the pitches of notes are pre-set before the act of performance) and intonation (applying particularly to singers, where the pitches of notes are controlled moment-to-moment by the performers, and to non-keyboard instruments), although there is of course a degree of overlap between the two concepts.

Our conclusion is that, in "normal" musical performance situations, it makes more practical sense to treat intervals as approximate distances on a one-dimensional scale than as ratios. When keyboards are tuned to the 12-EDO scale, the perfect 5th is tuned to an interval of approximately 700 cents; when tuned to 19-EDO it's approximately 694 cents. Such tuning is a feature not only of our digital harmonium, constructed for use in the Sufi Fragments, but is also related to some extent to historical precedent, in 17th-century harpsichords with split keys, such as those now found in the Reid Museum of Musical Instruments in Edinburgh, ie the 7 "white notes" (F C G D A E B), plus (up to) 12 "black notes", selected from F♯, C♯, G♯, D♯, A♯, E♯, B♯, and F♭, C♭, G♭, D♭, A♭, E♭, B♭. In fact, the historical precedents for 19-EDO tuning were one of the things which prompted the composition of the Sufi Fragments in the first place, although, of course, 19-EDO is probably our contemporary approximation to 17th century precedents with 19 tones per octave which were embodied by those instruments, where the distances between the consecutive notes of the chromatic scale were probably not entirely equal: something like a version of Extended Mean-tone Tuning was probably used on such instruments.

2: A scientist's story.

This paper came into being as a consequence of some conversations between the two authors over a period of some years about the purposes behind (specifically) the Conference on Interdisciplinary Musicology (CIM), and (more generally) a raft of attitudes which led to the creation of CIM. CIM was an initiative of (in particular) the second author (RP), back in 2004. He noticed that some questions concerning the way human beings interpret music had been investigated by scientists as well as by scholars in the humanities (including researchers in what — in France, especially — are sometimes called "the human sciences": psychology, sociology, anthropology, philosophy, literature, linguistics and related disciplines) and that in addition, practising musicians (both performers and composers) often had deeply rooted beliefs of their own concerning these questions, even though they may not have published any writings or promulgated any theories per se on these topics.

In principle, CIM requires authors from two different backgrounds to collaborate on the papers they present: science and humanities, science and practice or humanities and practice. Academics tend to say that they have been doing this for many years, but often the claim can be rather thin. We know of authors who have written papers, and then rung up a colleague and asked him/her to add his/her name to it! The result in such cases is not so much a collaboration as an endorsement of the work of the author by his/her colleague. And sometimes, it must be admitted, later CIM organisers have been less than fanatical about insisting on this dual-author qualification. Moreover, as the editor's prefatory comments to this volume of SMR point out, there has risen in recent years, a generation of scholars with qualifications in both science and humanities and even in practice as well.

Indeed, in recent years, virtually all educational institutions claim to promote inter-disciplinarity via their "mission statements" and similar paraphernalia. But such claims are, in many cases, rather hollow, calling to mind George Orwell's sarcastic comments on another universally-endorsed word, democracy:

"In the case of a word like democracy, not only is there no agreed definition, but the attempt to make one is resisted from all sides. It is almost universally felt that when we call a country democratic we are praising it: consequently the defenders of every kind of regime claim that it is a democracy and fear that they might have to stop using the word if it were tied down to any one meaning." (Orwell 1961:337-351)

Another problem is that what counts as evidence in one field may not do so in quite the same way in another. A scientific theory can, in principle, be subjected to controlled experiments, but that is hardly practicable when it comes to a Theory of History, Politics, Art or Society. Nevertheless, members of Society at Large still hold theories of History, Politics, Art and many other topics with scarcely less seriousness that they hold scientific theories of various kinds to be true: at least in the sense that they act in accordance with them. The Academy is still as partitioned into disciplines more or less as it was 50 years ago, even though Faculties are now often amalgamated into larger categories, such as Schools. But perhaps this has as much to do with the corporatisation of the contemporary Academy, and the pulling of it together under tighter financial and managerial control as with the promotion of inter-, multi- and trans-disciplinarity.

Following some conversations of this kind, the two authors published a long article in the Journal of Interdisciplinary Music Studies: (JIMS) (Parncutt and Hair 2011) outlining some thoughts on how we might reconsider the age-old questions of consonance and dissonance in music in the contemporary context. We put forward the view that many factors contributed to the impression in the minds of human beings (in at least our Western culture) of a sense of consonance and dissonance: ie that the consonance/dissonance dichotomy is a multi-factorial phenomenon, based on combinations of numerous additional dichotomies which are in some degree analogous. This does not, of course, deny that in various musical contexts, different dichotomies may not be accorded primary or secondary importance, and that, in respect of both ontology and epistemology, the consensus (to the extent that there is any) has changed over the centuries.

In what follows, we lead on from these thoughts, again prompted by the contemporary context, to what is perhaps an even more basic topic: the ontology of intervals. Again, we shall argue for the involvement of many factors and the important role of context. Nevertheless, we shall argue for the view that intervals are primarily distances along a one-dimensional scale, whereas for centuries, since the time of the semi-mythical figure of Pythagoras, and following the work of Helmholtz in the nineteenth century, musicians have tended to conceive of intervals as ratios between frequencies. In part, we shall also argue that the distances involved are approximate, and that consonance and dissonance play a role creating the extent and limits of categories of distance, and discuss in passing how composers and performers have for centuries "pushed at the boundaries" of these categories.

We shall also argue that there are many reasons why our view is not universally shared in contemporary culture, and that these include the nature of modernity itself. This is perhaps partly because modernity itself is also comprised of a bundle of features, some of which are in conflict with one another. Modernity is usually seen as a product of what has become known as The Enlightenment, in the second half of the eighteenth century, albeit the social circumstances which brought it about had been coming into being for generations or even a couple of centuries before

that time, and historical developments since have modified it to some extent. The mot d'ordre of the French Revolution (liberté, egalité, fraternité) is often taken as one of the the touchstones of The Enlightenment, but these ideals are themselves in conflict, and may not be the most appropriate — let alone the only — relevant mot. In the early twenty-first century — for example — we live in what some have called the age of terrorism, which is at the same time the age of human rights, the age of democracy, which is at the same time the age of authoritarian regimes and dictatorships, the age of materialism, which is at the same time the age of fundamentalisms of all kinds: economic, social and political as well as religious.

Obviously these questions may seem rather remote from such a specific question as the ontology of musical intervals, but we will suggest that they are amongst the factors which create mindsets and ways of thinking which affect the way people think and act about all sorts of topics, specific and general.

Our modest suggestion, enacted under the umbrella of CIM and JIMS, is that a useful way of taking the discipline of music studies forward, would be for agents of research in science, humanities and practice to work in collaboration in reality as well as in propaganda, and we offer here a small soupçon of such a project. The first author (GH) is involved in practice (composing, directing performances — mainly of singers — and creating, recording and editing music with technological resources) and writing (mainly about contemporary composers and music theory), all in 2016. The second author (RP) is a scientist who, for 30 years up to the present day, has written many books, chapters and articles and given quite a few conference key-note addresses about the way in which the human mind and body interprets musical sounds and structures.

3: Ontology and Epistemology in context(s): historical, philosophical, sociological, political, anthropological ...

Neither of the two foregoing accounts of specific, practical, personal experiences should lead us to lose sight of the fact that many widely-shared beliefs about music are much influenced by factors other than empirical observation — in the form of historical, philosophical, sociological, political and perhaps anthropological ideas — where the issues may not be primarily those of truth, falsehood and the advancement of knowledge as much as the propagation of aesthetic, ethical, moral, cultural and intellectual values and mores: in which circumstance it is difficult to imagine how it would ever be possible to navigate around ideological implications.

That is to say that all of the aforementioned academic disciplines — history, philosophy, sociology, politics, anthropology — carry within them large consignments of conventional wisdoms embodying such ideological implications, which — on the one hand — enable us to negotiate our way through complex forests of competing ideas, but — on the other — constitute an influence which it is difficult to slough off, even when times change and the usefulness of some established conventional wisdoms has withered away and alternative ones are gaining traction.

Musicologist Leonard Meyer (1994: 328) expressed the give and take between empirical observation and ideological values in terms germane to our topic when he wrote that "... realms external to music affect the history of music by changing the beliefs and attitudes ... that establish the values and goals in terms of which composers and performers, patrons and audiences choose particular musical means." (Meyer 1967, 1994).

Richard Taruskin (2010:340) put this thought even more strongly: "... we all — composers, performers and listeners alike — have strongly vested material, moral, intellectual and spiritual

interests that we make our music serve."

For example, in the fourth century BCE, philosopher Plato one of the foundational thinkers of Western civilisation, believed that the physical phenomena of everyday objects and human existence were an imperfect realisation of an ideal perfect world lying beyond them, and that human existence should endeavour to attain to that perfect world. A glimpse into that utopia was provided by the world of numbers, underlying every aspect of God's perfect world, including the celebrated Harmony of the Spheres, of which humankind's music was an imperfect realisation.

Nicomachus of Gerasa (circa 60CE – circa 120CE) was a neo-Pythagorean mathematician of late antiquity, whose Manual of Harmonics (a primer or handbook of received thought, rather than a work of original ideas) was one of the earliest influential works of music theory written in the period after Aristoxenus (circa 375BCE – 325BCE) but drawing on traditions from the earlier ancients, going right back to Pythagoras (428/427BCE – 348/347BCE). In particular, there is a Pythagorean tradition transmitted through such writings as Plato's Timaeus: a tradtion which focuses on the mathematical interpretation of nature. As Kahn says: "... the Pythagorean-Platonic tradition of the Timaeus has, in a sense, retained its vitality down to our own day..." (Kahn 2001:153).

One numerical notion considered to embody such utopian aspirations imperfectly to represent God's perfect world was the so-called tetractys, sometimes called the "mystical tetrad": a geometrical figure of points grouped in rows of 1, 2, 3 and 4 (cf Figure 2) ...

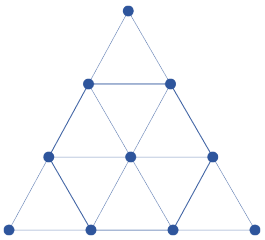

Figure 2: The tetractys symbol

... a structured enumeration of the first ten integers, and which was thought implicitly to represent the underlying mathematical connectedness of the structures and forms of all things, from the workings of the smallest element in the nature to the largest: a mystical representation of the perfect order of the universe. Dantzig even quotes what is thought to have been an ancient prayer to the tetractys, composed by Pythagoreans:

"Bless us, divine number, thou who generated gods and men! O holy, holy Tetractys, thou that containest the root and source of the eternally flowing creation! For the divine number begins with the profound, pure unity until it comes to the holy four; then it begets the mother of all, the all-comprising, all-bounding, the first-born, the never-swerving, the never-tiring holy ten, the keyholder of all." (Dantzig 1930, 2007:42).

To most of us in the twenty-first century, such formulations undoubtedly sound like hocus-pocus, but, perhaps the tetractys can be understoood simply as a symbol for the perfection of the universe which numbers were felt to represent. Indeed, the idea that there must be a basic principle underlying the order of the universe is still with us in the form (for example) of hypothetical models of the so-called Grand Unified Theory, unifying the four physical forces of gravity, electro-magnetism and the strong and weak nuclear forces into a single, all-encompassing, coherent theoretical framework

of physics that fully explains and links together all physical aspects of the universe.

Some basic and persistent doctrines in the discipline of music-theory can also be traced back to ancient ideas such as those of Plato and Pythagoras: often as transmitted through medieval culture, as, for example, via doctrines found in the writings of Boethius (480 – 524CE), one of the music-theory's founding fathers, who is usually considered to be a neo-Platonist.

Leonard Meyer (1967, 1994), writing nearly a millennium and a half after Boethius, was nevertheless able to advance a reason why ancient quasi-mystical numerological formulations have sometimes persisted in music-theoretical thinking even into the late twentieth century: namely that utopian ideologies have continued to hold sway over contemporary thought about many topics, including music-theoretical ones, even in the post-Enlightenment era, albeit for very different reasons, and with very different results than in ancient and medieval times, indeed in pre-Enlightenment times generally.

"In the early millennia of Western culture — say through the Middle Ages the present was judged in relation to a golden past: an age of uncorrupted Eden, of peerless gods and heroes, or of incomparable civilization … And that exemplary order was the authorative source of beliefs and knowledge, values and goals … A great revolution in man's conception of the world took place when, between the fifteenth and seventeenth century, the golden age moved from the past to the future … Nurtured on the successes of science and technology, the Idea of Progress became an article of faith in the eighteenth and nineteenth centuries … The more change, the sooner utopia, whether a perfect democracy, a classless society, or a world without want or war … All realms of human activity — the sciences, the arts, philosophy, history and religion — were future-oriented and goal-directed … In short … one of the most consequential beliefs in any culture is its beliefs about the relationship of the past to the present and especially of the present to the future.(Meyer 1967, 1994:328 – 330).

4: The influence of historical, philosophical, sociological, political, anthropological forces on empirical observation

Meyer's is not at all an exceptional viewpoint, but a widely-held interpretation of the trajectory of modern history, which derives from contemplating the (not-so-obvious) implications of a (perfectly-obvious) fact: namely, that the history of the past is — in principle, and increasingly, as the techniques of empirical investigation of evidence and sources gradually improve — relatively knowable, but totally unchangeable, whereas the history of the future is — it has often been imagined, in the two-and-a-half centuries of the post-Enlightenment Age — unknowable, but potentially changeable according to the will and desires of humankind.

Most authors also customarily identify the Enlightenment as the "authoritative source of beliefs and knowledge, values and goals" in modern times, taking it to refer to a bundle of features characterised generically as Modernity, or — when speaking of works of art — as Modernism, and the process of bringing Modernity and Modernism to a state of authority and influence as Modernisation: a bundle of features which usually includes demoncratisation, urbanisation, industrialisation, secularisation, rationalisation and others: all of them essentially based on the pride of place in modern life accorded to science and technology.

The characteristic philosophical view of post-Enlightenment modern history was encapsulated in a famous maxim of the early post-Enlightenment years, penned by the characteristic philosopher of those times, Georg Wilhelm Friedrich Hegel (1770-1831), an exact contemporary of Beethoven: "The history of the world is none other than the progress of the consciousness of Freedom" (Hegel 1840,

1900:19), albeit qualified (in the same volume) by : "When liberty is mentioned, we must always be careful to observe whether it is not really the assertion of private interests which is thereby judged." (Hegel 1840, 1900:430).

In our own time, something of this dichotomy persists. Cultural historian and literary scholar Terry Eagleton is surely correct to characterise the Enlightenment as a "mighty current of human emancipation" (Eagleton 2010:22). Nevertheless, a much more cautious Hegelian — philosopher Roger Scruton — proposes that "It is reasonable to believe that science, in the right conditions, will continuously advance ... superstition lies in the belief that what is true of science, is true of every other human endeavour — of politics, morality, religion and art" (Scruton 1987:202-203).

Hegel thought that the conjunction of his maxims implied that post-Enlightenment society should evolve from reciprocity between individual self-interest and three principal institutions of his day and ours: the family, civil society and government. However, later thinkers, especially those known as The Young Hegelians, drawing inspiration from the Jacobins of French Revolutionary times and their slogan liberte, egalite, fraternity, beginning with Marx and Feuerbach, and persisting through the later communist states of west and east, and 20th-century philosophical traditions, through the so-called Frankfurt School, especially Adorno, to the French philosophical tradition now known ironically as the marxisant tradition (a tradition of priviledged middle-class Marxist academics theorising on behalf of the prolatariat) — enduring down to the death of Deleuze in 1995, and perhaps even beyond — continued to believe that society could — and should — be remade from the ground up, by enlightened, but coersive, top-down planning of one sort or another.

A 20th-century tradition we might describe as one of musical Young Hegelians came to believe that it was historically necessary and inevitable that music, being made in the image of society, could and should likewise be remade from the ground up, by enlightened individuals with visionary insight. For the most part, this attitude came about in music rather later than in scientific, philosophical and other broader fields of intellectual endeavour, mostly in the 20th century, in practices we might call solipsistic materialism: the importation into music of any sound practitioners know how to produce, and the importation of ideas about music, as well as musical ideas.

Musicologist Karol Berger has pointed out that music historiography stands in something of an eccentric position in relation to the consensus of other disciplines:

"The view that the late eighteenth century marks the arrival of modernity may be a commonplace amongst historians of politics, economics, society, culture, literature and art, but, with the significant exception of those influenced by Adorno, it is not widely shared by music historians ... In the standard usage of our music history texts, to say nothing of our concert life, twentieth-century modernism is the only 'modern' music there is" (Berger 2000:5).

Consequently, it is to the 20th century, especially its second half, that we must look particularly for the majority of utopian attempts to remake musical culture from the ground up, along lines parallel to the thinking of the earlier philosophical Young Hegelians.

Schoenberg famously proposed to Josef Rufer, concerning his discovery of the technique of "composing with twelve tones related only to one another", that "I have made a discovery which will ensure the supremacy of German music for the next hundred years" (Stuckenschmidt 2011:277). In the optimistic post-war years following the demise of Nazism, Pierre Boulez criticized Schoenberg's radical musical language for not being radical enough; for him, it retained too many rhythmic and formal concepts from the classical tradition: the very features which enable most listeners to extract some degree of coherence from it. John Cage's ideas for "liberating sound" reflect an even more

radical attempt to remake music, leading — in late 20th-century academia — to the reconfiguring of compositional practice as a form of research, or research-equivalent activity.

However, even Boulez, near the end of his life (in his interview on receipt of the Kyoto Prize in Japan in 2009) insisted that this attitude (music as a form of research) was wrong-headed (Boulez 2009). Literary scholar Lionel Trilling (Trilling 2008) thought it was also wrong in relation to literature, not because there is anything impossible about it per se, but because the social situation has changed since Hegel's day — and even since Adorno's — in almost every respect: everybody is now educated within The Academy to approach the making of art and music as "contrarian" (Trilling's term): a solipsistic activity whose implied duty is to place itself somewhere along the social spectrum from idiosyncratic, daring and path-breaking to transgressive, shocking, outrageous, threatening and dangerous, and whose practitioners are thus able to self-identify themselves to Society at Large as Heroic Resisters against the inundation of society by Mass Culture,

When Wagner wrote The Music of the Future in 1860, the shape of his harmonic thinking influenced much of the subsequent course of music for two generations following. But now that 50 years of 20th-century avant-gardism have elapsed, it is abundantly clear that 99 per-cent of avant-garde practice leaves little trace on subsequent generations. The term avant-garde has mutated into an "epithet of self-identification", rather than a predictor of the shape of the future. Harvard sociologist Daniel Bell (1919-2011) went so far as to describe such utopian fantasies as the dream of remaking of society through visionary individual action as "the meglomania of self-infinitisation" (Bell 1976, 1996:49): an epithet which could equally apply to most agendas for the remaking of music from the ground up as well.

Of course, catastrophic social, political, financial and other events in the early 21st century have cast serious doubt on virtually all such utopian ideas, political, economic, social and musical, which may thus be now in decline. The words of historian David Christian, two centuries after Hegel, in 2005, could be construed as bringing Hegel's maxims up to date:

"If I had to sum up the twentieth century, I would say that it raised the greatest hopes ever conceived by humanity, and destroyed all illusions and ideals" (Christian 2005:323).

The construal of intervals as precise mathematical ratios exemplifies a utopian concept sometimes described as the Nirvana Fallacy: a particular manifestation of Modernity, which, like other utopian projects of the post-Enlightenment era — including the total remaking of music — involves the putting of perfect, imagined, unrealistic, idealized solutions, in the place of achievable real things. Essentially, the Nirvana fallacy is form of the misapplication of science: of the fact that modern science and technology permit us to handle frequency ratios very easily and precisely, despite the limited relevance of ratio measurements to the actual making of music.

These are examples of a particularly Modern (post-Enlightenment) form of utopianism, which, as we noted in earlier quotes, Meyer describes as imagining a Golden Age, not in the remote past (as in the case of the Weltanshauung of the philosophers of the ancient world or the Middle Ages), but as something still to come, to be brought about by visionary thinking in the present; in sum: a refusal to let how things actually are stand in the way of how things really ought to be (Meyer 1967, 1994).

It is again Karol Berger who offers a more balanced view of such utopian attitudes, in particular relation to music:

"At mid-twentieth century, one tended to attibute to the undermining of tonal and metric framework fifty years earlier the fundamental importance which made all preceding changes in musical

history look pale by comparison. Today we are more inclined to see the Schoenbergian and Stravinskian expressionisms as contingent stylistic developments rather than inevitable revolutions and hence more likely to consider them as episodes within a longer story" (Berger 2005:7).

The lesson we propose from Berger's sober assessment — and David Christian's — is a modest one, which is nevertheless important: the need for practising musicians and scholars in the humanities to consider more seriously what scientists with a background in empirical research can contribute to the way we understand musical materials, including intervals, alongside what we may learn from music theory, practice and history, together with general and cultural history, philosophy, sociology, politics and anthropology.

The so-called Common Practice Tradition of western music has always been a collective practice: brought about by interaction of practitioners with Society at Large, and by the interaction of present practices with those of generations long since dead, plus — to be sure — a modicum of "boundary pushing" in directions along which individuals in the present anticipate that current norms might be modified or expanded,

As Roger Scruton put it: "A musical tradition is not an invention ... No single person created it ... any more than a single person could discover a language, [a phenomenon which] has no first principles, no definitions, no a priori system" (Scruton 2016:399-407).

5: Juataposing Historical, Philosophical, Sociological, Political and Anthropological issues with Empirical Research

Stevens (1946) presented a general theory of psychological scales and measurement that had important implications for psychoacoustics. It also has interesting implications for music theory, and is further evidence against the Pythagorean concept of musical interval.

Stevens's concept of "scale" differed from the musical concept. He considered the problem of perceiving points on a scale relative to each other. For example, is a 100-watt light globe twice as bright as a 50-watt light globe? The answer is no, even if the 100-watt globe is producing twice as much light power, because perceived brightness is not proportional to light power. The musical concept of scale refers to something quite different: a collection of tones of nominally fixed pitches, used for making music. Stevens' general psychological discussion of "scale" is nonetheless relevant for a psychological understanding of musical scales.

Stevens asked whether it possible to measure human sensation, and defined measurement in a very general way as "the assignment of numerals to objects or events according to rules" (p. 677). He then defined four kinds of scale: nominal, ordinal, interval and ratio. Musical scales can be of all four types. They are nominal scales (because scale steps have names: do re mi etc), ordinal scales (because scale steps are in a fixed order), interval scales (because intervals have the same size at different places in the scale), and ratio scales (because two octaves are perceived as twice as big as one octave, two semitones as twice as big as one).

Psychological research has not been supportive of the Pythagorean concept of musical intervals as ratios. Experiments have revealed consistent stretching of musical octaves and other larger intervals in performance relative to simple ratios, and compression of smaller intervals. To our knowledge, no researcher has ever discovered a recording of a music performance by voices or instruments with real-time pitch adjustment (such as typical wind or string instruments) that consistently conforms to either Just or Pythagorean intonation.

Burns and Ward (1978) presented pairs of tones to musicians and asked them to name the

intervals. The musicians were good at this task and seldom made errors. The intervals in the experiment were mistuned by as much as a quartertone, but the musicians reliably quickly named them relative to the twelve categories of 12-EDO. For example, they quickly identified an interval of 350 cents as either a minor third or a major third, and an interval of 450 cents as either a major third or a perfect fourth. The authors concluded that musical pitch perception is categorical. Each perceptual category has a label corresponding to a chromatic scale step and a width of about one semitone. The empirical evidence on intonation suggests that musical intonation is acceptable if listeners can correctly categorize the tones in the chromatic scale, that is, if pitches lie within those category boundaries. Under good conditions for performing frequencies exactly and perceiving pitches exactly, the category boundaries for intonation become smaller than the category boundaries for interval recognition. One may recognize an interval as a major third if it lies between 350 and 450 cents, but under ideal conditions it may be considered "in tune" if it lies between 380 and 430 cents.

We can imagine a listener to any such music, in any style or genre. The music suddenly stops and she hears a pure tone with randomly selected frequency in the central musical range. In general, she will hear this tone relative to the prevailing diatonic and chromatic scales, as one of the chromatic steps (the closest) or as a natural or shifted diatonic step. Unless she has absolute pitch, or good relative pitch plus knowledge of the key in which the music was playing, she will not say "aha, that was a D♭" or "aha, that was the fourth scale degree in A♭ major", but she will hear the tone as if it were one of those things. If the tone had been tuned differently, but was still within a quartertone of a D♭, she would hear it as the same tone and may not even be aware of the mistuning. She would assume that the musicians had intended to play D♭, even if the tone was clearly out of tune.

This thought experiment also explains why pitch is perceived categorically. In most music, mistunings of a quartertone or even a semitone are commonplace. The ear must be able to tolerate mistunings of this order, otherwise musical appreciation would be impossible. Mistunings of partials within harmonic complex tones are similarly commonplace; a familiar example is the stretched harmonic series in every piano tone (Martin and Ward 1961).

That can explain why partials within a harmonic complex tone can be mistuned by a quartertone or even a semitone without being separately noticed (Moore et al. 1986). These findings imply that the tuning of partials within complex tones is also perceived categorically: individual audible partials are perceived as either belonging or not belonging to a given harmonic complex tone. The empirical literature on the categorical perception of musical pitch implies that intervals do not sound as far out of tune as they really are. We are remarkably good at ignoring mistuning in performance and guessing the pitch a performer is aiming for, as if our perception were shifted toward the intended pitch of each tone (zurecht hören: Bruhn (1994), Fricke (1988) and Kurth (1931)). It cannot be otherwise if the results of studies such as Devaney et al. (2012), Kopiez (2003) and Rakowski (1990) are correct. Categorical perception is like a filter that assigns out-of-tune intervals to familiar categories, as if they were in tune. This is a saving grace for all amateur choirs and orchestras, whose intonation is regularly out by a quartertone or even a semitone.

Categorical perception of musical pitch contradicts Pythagorean theory, in which musical intervals are held to have single, exact sizes. The concept of categorical perception allows us to define a musical interval in a quite different way. Each musical interval label corresponds to a continuous range of interval sizes. Any interval in the range 350 to 450 cents may be perceived as a major third. Plus or minus a quartertone might seem like a very big range of uncertainty, but that is essentially

what the experiment of Burns and Ward and others established (eg Halpern and Zatorre (1979), Siegel and Siegel (1977)). We are aware of no good empirical studies of intonation in performances of typical music (rather than music in which tuning is most exact) by choirs, string quartets and so on. Such an empirical study would carefully address the measurement issues addressed above and apply appropriate statistical methods to the evaluation of large numbers of measurements from diverse musical contexts.

So we may ask why modern Pythagoreans cling to beliefs about ratios in spite of the overwhelming empirical evidence to the contrary — for example, the evidence that intonation varies widely in music performance and often deviates systematically from ratios. Several possible reasons can be identified. First, we tend to idealize music because of the wonderful emotions it evokes, seeking a beautiful theory to account for a beautiful phenomenon. We associate positive emotions with tones that sound exactly in tune, and hence harmonious with other tones. Music is a form of culture that shares utopian features with the Platonic and neo-Platonic thought of the ancient and medieval worlds, but it is not the only one.

A second reason why many still cling to Pythagorean beliefs is that they have successfully explained aspects of music for very long historical periods. In the history of science and ideas, we can observe a general reluctance to give up previously successful theories. Like Kuhn's paradigms in the history of science (Kuhn 1962), Pythagoreanism may only be overthrown when its failures become intolerable and the successes of an alternative become compelling, just as classical mechanics was overthrown by quantum mechanics and relativity. Implicitly such failures are embodied in the reaction of the singers to the 12-EDO and 19-EDO examples with which we started this discussion.

As an approximation, however, the Pythagorean approach will always be useful, just as classical mechanics is still useful. From a scientific viewpoint, a paradigm change in the concept of musical interval may already have happened; but some Pythagorean concepts are still considered valid, at least in certain contexts or domains, for example by humanities scholars or musical practitioners. Musical practice is constantly changing in accordance with changes in the economic, sociological, philosophical, historical, political, ideological and other circumstances; humanities disciplines are consequently "devoted less to the advancement of knowledge than to the propagation of moral and intellectual values" (Scruton 2007:3). Thus, the problem may lie in difficulties of interdisciplinary communication and the mutual recognition of knowledge across disciplinary boundaries as recognized for example by Snow (1959).

Third, we tend to exaggerate the "exactness" of the so-called "exact sciences". Physics is often thought of as an "exact science" because it is dominated by mathematical theory. But mathematics is also an excellent tool for dealing with inexactness. One approach is the order-of-magnitude estimate: the size of a quark is less than $O(10^{-18})$ m; the weak nuclear force operates over a range of $O(10^{-14})$ m; the temperature of a supernova expansion is $O(10^{10})$ Kelvin; there are $O(10^{14})$ cells in the human body; and there are $O(10^{80})$ fundamental particles in the universe. Music theory is often considered mathematical and therefore exact, but in fact there is no clear relationship between mathematics and exactitude. Applied mathematics varies along a spectrum from very exact to very approximate. The number ratios that correspond to musical intervals also lie somewhere along that spectrum.

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